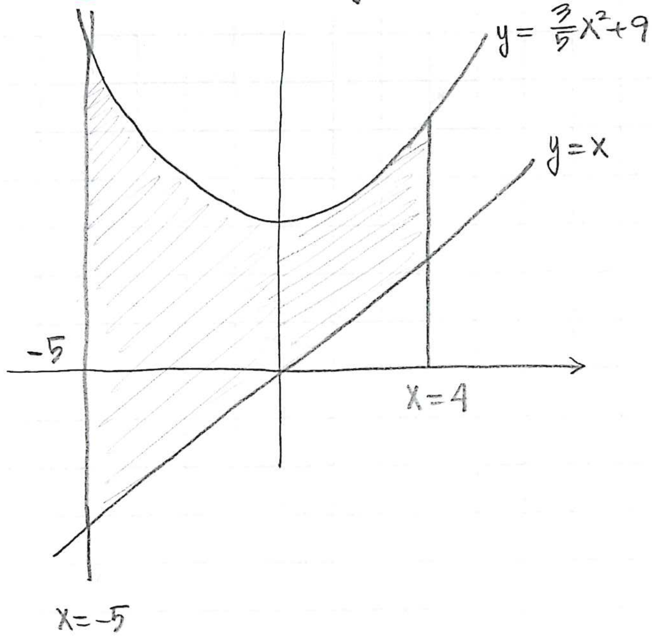
# 5.1. Areas Between Curves.

① Area b'dd by the curves $y = \frac{3}{5}x^2 + 9$, $y = x$, $x = -5$, $x = 4$.




$$A = \int_{-5}^{4} \left(\left(\tfrac{3}{5}x^2+9\right) - (x)\right) dx$$

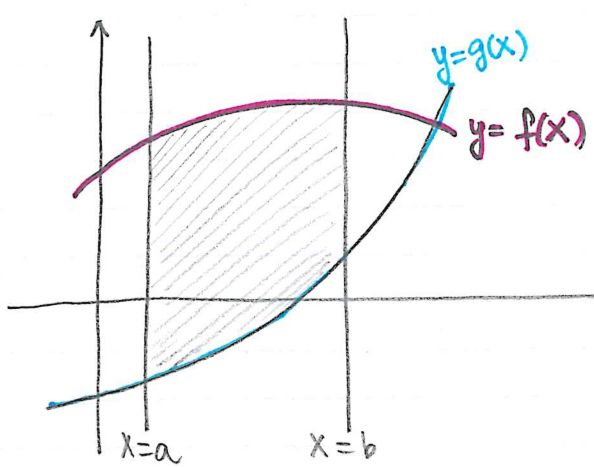
$$= \left( \frac{3}{5}\frac{x^3}{3} + 9x - \frac{x^2}{2} \right)\Big|_{-5}^{4}$$

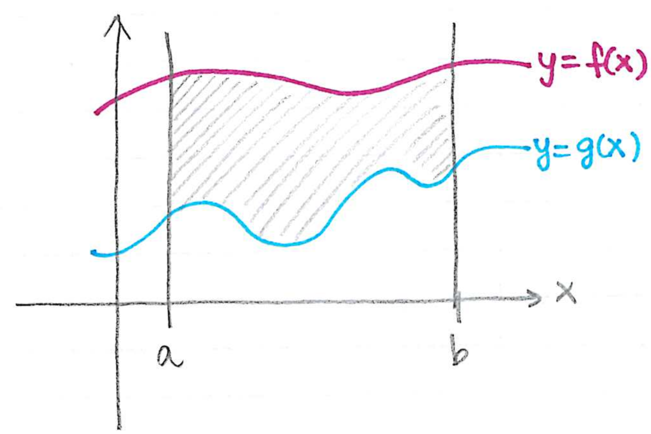
$$= \left(\frac{64}{5} + 36 - 8\right) - \left(-25 - 45 - \frac{25}{2}\right)$$

$$= \frac{64}{5} + \frac{25}{2} + 98$$

The area A of the region bounded by the curves $y = f(x)$, $y = g(x)$, and the lines $x = a$, $x = b$, where $f$ & $g$ are continuous and $f(x) \geq g(x)$ for all $x \in [a,b]$, is:

$$\boxed{A = \int_a^b [f(x) - g(x)]\, dx}$$



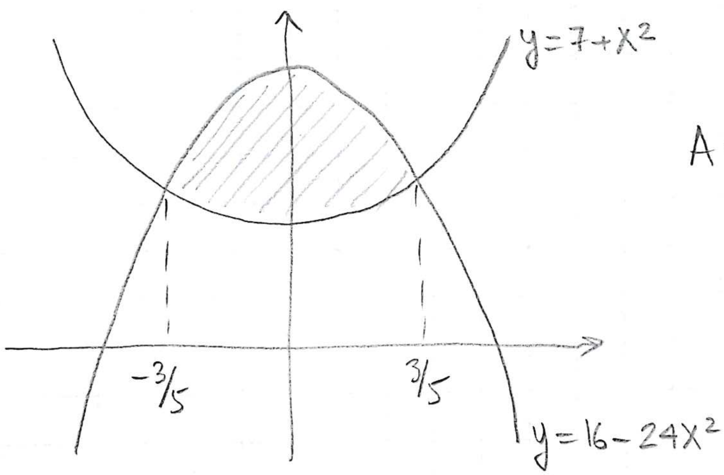



(2) Region b'dd by $\underbrace{y=16-24x^2}_{\text{concave parabola w/ roots } \pm\sqrt{\frac{16}{24}}=\pm\sqrt{\frac{2}{3}}}$ and $\underbrace{y=7+x^2}_{\text{convex parabola w/ no real roots}}$.

<u>Intersection?</u>

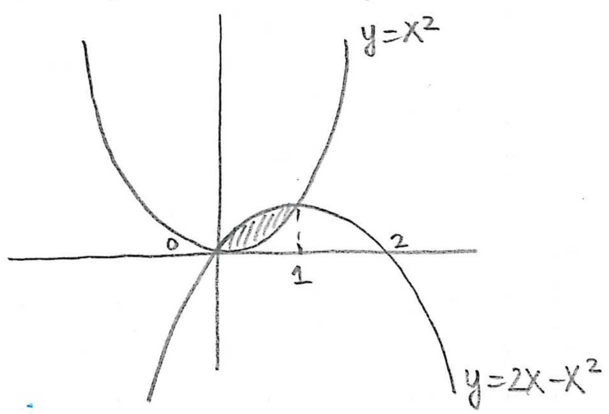
$$16-24x^2 = 7+x^2$$
$$9 = 25x^2$$
$$\pm\frac{3}{5} = x \Rightarrow y = 7+\frac{9}{25} = \frac{184}{25}$$
$$= 16-24\cdot\frac{9}{25} = \frac{184}{25}$$

$\Rightarrow$ Points of intersection: $\left(-\frac{3}{5}, \frac{184}{25}\right), \left(\frac{3}{5}, +\frac{184}{25}\right)$.




$$A = \int_{-3/5}^{3/5} \left((16-24x^2)-(7+x^2)\right)dx$$
$$= \int_{-3/5}^{3/5} (9-25x^2)\,dx = \frac{2700}{375}$$

(3) Region enclosed by parabolas $y=x^2$ and $y=2x-x^2$.




Points of intersection:

$$x^2 = 2x-x^2$$
$$0 = 2x-2x^2$$
$$0 = x(1-x)$$
$$x=0,\ x=1$$

$$A = \int_0^1 \left((2x-x^2)-x^2\right)dx$$
$$= \int_0^1 (2x-2x^2)\,dx = \left(x^2-\frac{2x^3}{3}\right)\Big|_0^1$$
$$= 1-\frac{2}{3} = \frac{1}{3}.$$

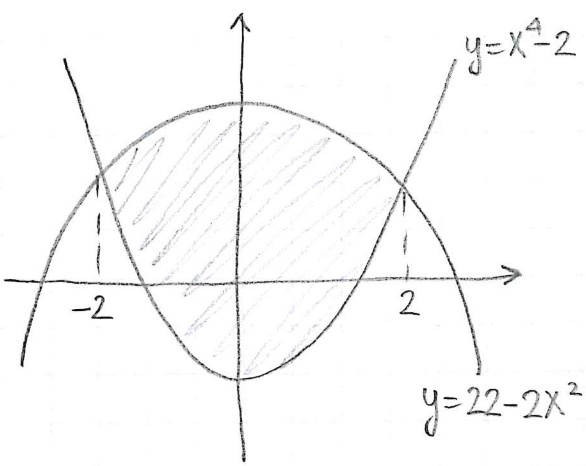
④ Region b'dd by $\underbrace{2x^2+y=22}$ & $\underbrace{x^4-y=2}$

$y=22-2x^2$
concave parabola w/ roots $\pm\sqrt{11}$

$y=x^4-2$ (looks like a parabola, has roots at $\pm\sqrt[4]{2}$)
$=(x^2-\sqrt{2})(x^2+\sqrt{2})$

Intersection: $22-2x^2=x^4-2$
$x^4+2x^2-24=0$
$\underbrace{(x^2+6)}_{\text{no real roots}}\underbrace{(x^2-4)}_{\text{roots @ } x=\pm 2}=0$

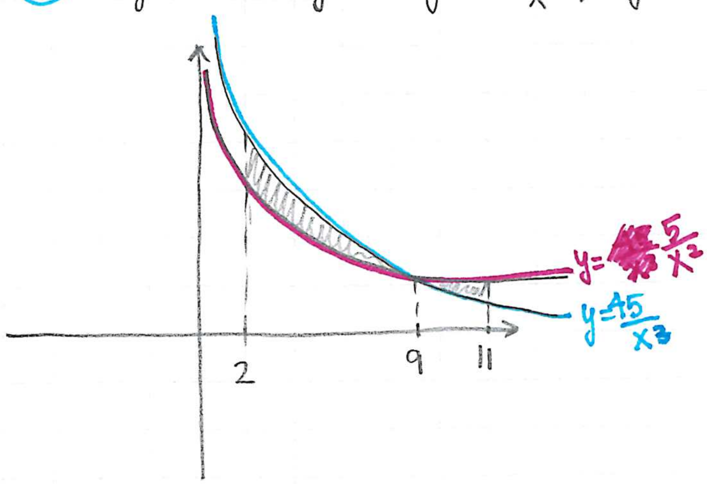
$(2,14),\ (-2,14)$




$$A=\int_{-2}^{2}\left((22-2x^2)-(x^4-2)\right)dx$$
$$=\int_{-2}^{2}(24-2x^2-x^4)\,dx$$
$$=\frac{1088}{15}$$

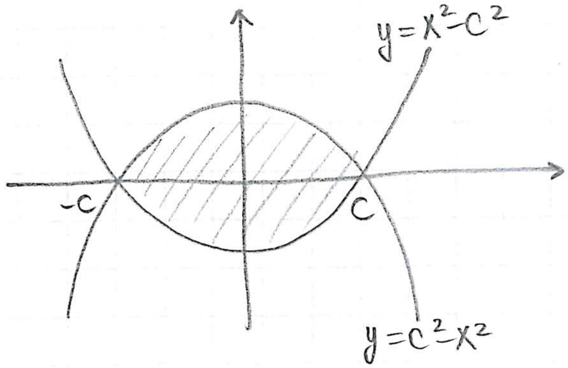
⑤ Region b'dd by $y=\frac{45}{x^3},\ y=\frac{5}{x^2},\ x=2,\ x=11.$

$$\frac{45}{x^3}=\frac{5}{x^2} \Rightarrow 9x^2=x^3 \Big\} \Rightarrow x=9$$
$x\neq 0$




$$A=\int_2^9\left(\frac{45}{x^3}-\frac{5}{x^2}\right)dx+\int_9^{11}\left(\frac{5}{x^2}-\frac{45}{x^3}\right)dx$$
$$=\left(\frac{-45}{2x^2}+\frac{5}{x}\right)\Big|_2^9+\left(\frac{-5}{x}+\frac{45}{2x^2}\right)\Big|_9^{11}$$

6) Find $c>0$ s.t. the area of the region b'dd by $y=x^2-c^2$, $y=c^2-x^2$ is 18.




$$x^2-c^2=c^2-x^2$$
$$2x^2=2c^2$$
$$x=\pm c$$

$$A=\int_{-c}^{c}\left((c^2-x^2)-(x^2-c^2)\right)dx$$
$$=\int_{-c}^{c}(2c^2-2x^2)\,dx$$
$$=\left(2c^2x-2\frac{x^3}{3}\right)\Big|_{x=-c}^{x=c}$$
$$=\left(2c^3-2\frac{c^3}{3}\right)-\left(-2c^3+2\frac{c^3}{3}\right)$$
$$=4c^3-\frac{4}{3}c^3=\frac{8}{3}c^3$$

$$\frac{8}{3}c^3=18$$
$$c^3=\frac{3\cdot 18}{8}=\frac{3\cdot 9}{4}$$
$$c=\left(\frac{27}{4}\right)^{1/3}$$

⑦ Area b/w graphs $x+2y=18$ & $x+6=y^2$

$y=9-\frac{x}{2}$ $\quad$ $x=y^2-6$

$x=18-2y$

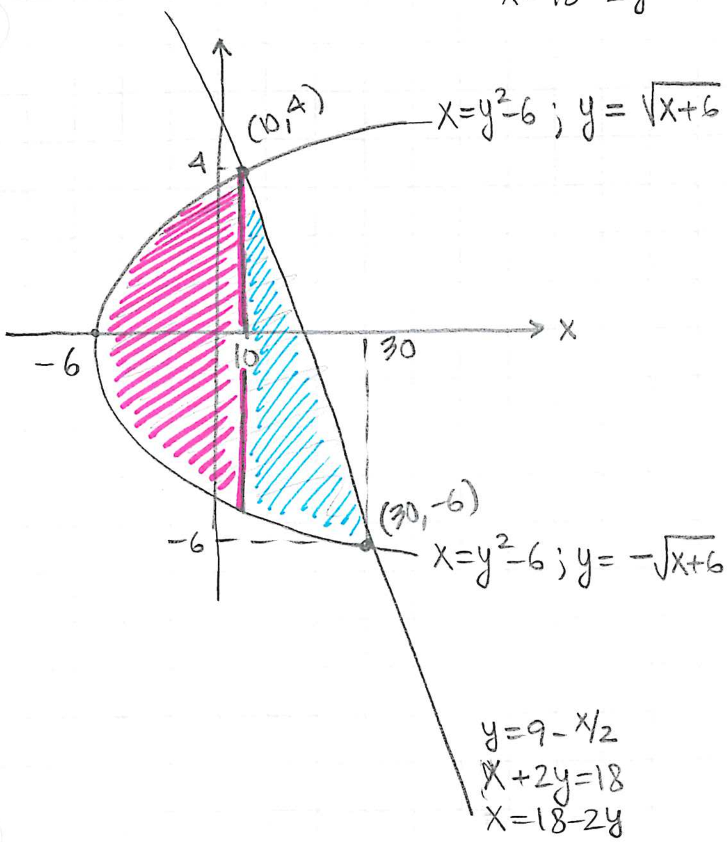
$18-2y=y^2-6$

$y^2+2y-24=0$

$(y+6)(y-4)=0$

$\underbrace{y=-6}_{x=30}, \underbrace{y=4}_{x=10}$

Pts.: $(30,-6)$, $(10,4)$




$$\int_{-6}^{10}\left(\sqrt{x+6}-\left(-\sqrt{x+6}\right)\right)dx+\int_{10}^{30}\left(\left(9-\frac{x}{2}\right)-\left(-\sqrt{x+6}\right)\right)dx$$

$$=\int_{-6}^{10}2\sqrt{x+6}\,dx+\int_{10}^{30}\left(9-\frac{x}{2}+\sqrt{x+6}\right)dx$$

OR directly on the y-axis: $\displaystyle\int_{-6}^{4}\left(\left(18-2y\right)-\left(y^2-6\right)\right)dy$

$$=\int_{-6}^{4}\left(24-2y-y^2\right)dy$$